

A new mathematical model to localize a multi-target modular probe for large-volume-metrology applications

*Original*

A new mathematical model to localize a multi-target modular probe for large-volume-metrology applications / Maisano, DOMENICO AUGUSTO FRANCESCO; Mastrogiacomo, Luca. - ELETTRONICO. - (2017). (Intervento presentato al convegno 11th AMCTM (Advanced Mathematical and Computational Tools in Metrology and Testing) International Conference tenutosi a Glasgow (UK) nel 29-31 August, 2017).

*Availability:*

This version is available at: 11583/2678947 since: 2017-09-07T13:29:00Z

*Publisher:*

None

*Published*

DOI:

*Terms of use:*

openAccess

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

(Article begins on next page)

## A new mathematical model to localize a multi-target modular probe for large-volume-metrology applications

D. Maisano<sup>†</sup> and L. Mastrogiacomo

*Dept. of Management and Production Engineering (DIGEP), Politecnico di Torino,  
Corso Duca degli Abruzzi 24, Turin, 10129, Italy*

<sup>†</sup>*E-mail: domenico.maisano@polito.it  
www.polito.it*

*Keywords:* Large-volume metrology; Multi-target probe; Real-time localization, Generalized least squares.

### Abstract

Typical industrial applications in the field of *Large-Volume Metrology* (LVM) concern dimensional verification and assembly of large-sized mechanical components, in which levels of uncertainty of several tenths of millimetre are tolerated [1].

LVM systems are usually equipped with *sensors* that perform local measurements of *distances* and/or *angles*. Even though the existing measuring systems may differ in technology and metrological characteristics, two common features are: (i) the use of some *targets* to be localized, generally mounted on a hand-held probe to localize the points of interest or in direct contact with the measured object's surface, and (ii) the fact that target localization is performed using the local measurements by sensors (e.g., through *multilateration* or *multiangulation* approaches).

Recent studies show that the combined use of LVM systems can lead to a systematic reduction in measurement uncertainty and a better exploitation of the available equipment [2]. Unfortunately, the sensors of a specific LVM system are usually able to localize only specific targets and not necessarily those related to other systems; this represents a relevant practical obstacle.

To overcome this obstacle, the authors have recently developed a new *modular* probe, equipped with targets related to different systems, and a *tip* in contact with the point of interest ( $P$ ), which allows to localize  $P$  in a single turn [2]. This probe has several innovative features, e.g. (i) the number and typology of targets can be varied depending on the specific application, and (ii) it can integrate additional inertial sensors able to provide additional data, i.e., two-axis inclinometer and compass.

The goal of this paper is to present a new mathematical/statistical model to localize the probe in measurements involving combinations of different LVM systems, i.e., equipped with sensors of different nature and metrological characteristics. The model takes the following factors into account:

- Relative position between distributed sensors and probe targets.
- Uncertainty in the position/orientation of any distributed sensor.
- Uncertainty in the local (angular and/or distance) measurements by distributed sensors, with respect to probe targets.
- Number, typology and relative position of the probe targets (and corresponding uncertainty) with respect to the probe tip.
- Angular measurements (and relevant uncertainty) provided by the inclinometer and compass embedded in the probe.

More precisely, an overdefined system of equations is linearized and solved through the method of *Generalized Least Squares* (GLS) [3], which can be used to give greater weight to the equations producing less uncertainty and *vice versa*. For the model to be viable, some parameters relating to the sensors in use should be known in advance, e.g., uncertainties in the position/orientation or local measurements; this can be done through *ad hoc* experimental tests or using manuals or technical documentation of the measuring systems. The proposed model can also be used to estimate the uncertainty in the probe localization: a covariance matrix can be determined by applying the *Multivariate Law of Propagation of Uncertainty* (MLPU) to the system of equations.

The model is automatable and could be a key tool to promote the combined use of LVM systems.

### Acknowledgements

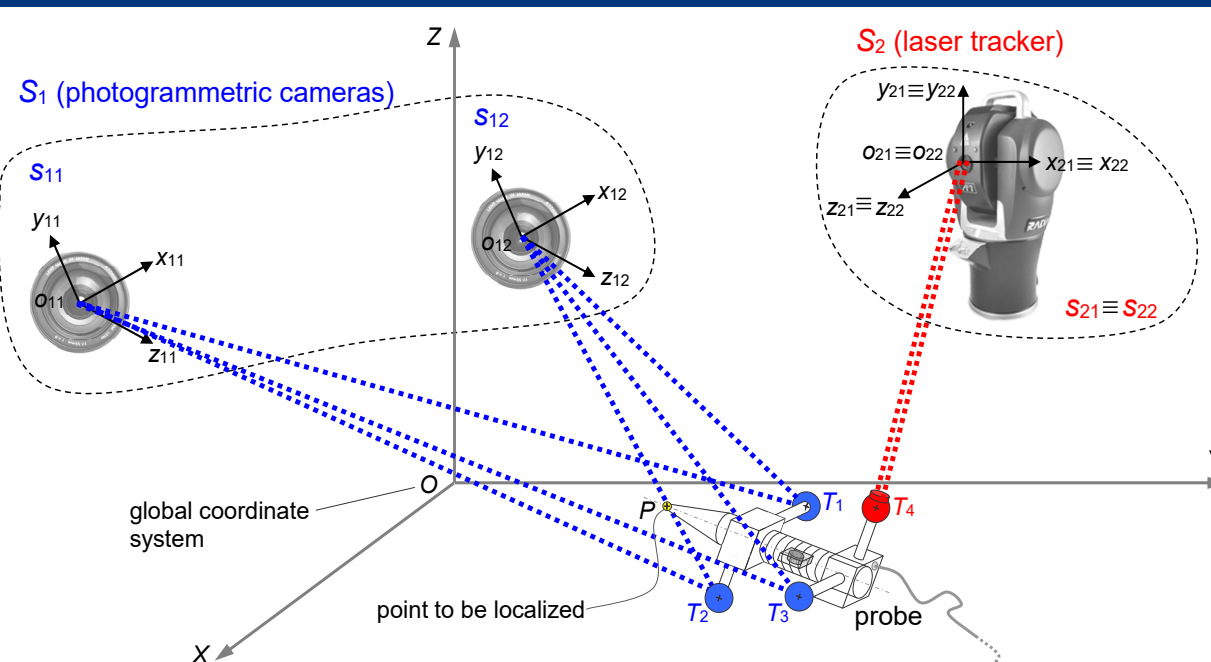
This research was supported by the project Co-LVM “Cooperative multi-sensor data fusion for enhancing Large-Volume Metrology applications” (E19J14000950003), financed by Fondazione CRT (Italy), under the initiative “La Ricerca dei Talenti”.

### References

1. Maropoulos, P.G., Muelaner, J.E., Summers, M.D., Martin, O.C. (2014). A new paradigm in large-scale assembly—research priorities in measurement assisted assembly. *The International Journal of Advanced Manufacturing Technology*, 70(1-4): 621-633.
2. Maisano, D., Mastrogiacomo, L., Galetto, M., Franceschini, F. (2016) Dispositivo tastatore e procedimento di localizzazione di un dispositivo tastatore per la misura di oggetti di grandi dimensioni, *Italian provisional patent application*, serial number 102016000107650, filed on 25/10/2016.
3. Kariya, T., Kurata, H. (2004) *Generalized least squares*, John Wiley & Sons, New York.

## Background

LVM systems ( $S_1, S_2, \dots, S_i, \dots$ ) are usually equipped with *sensors* ( $s_{i1}, s_{i2}, \dots$ ) that perform local measurements of *distances* and/or *angles*. Recent studies show that the combined use of different LVM systems – i.e., with sensors of different nature and metrological characteristics – can lead to a systematic reduction in measurement uncertainty and a better exploitation of the available equipment. The authors have recently developed a new modular *probe*, equipped with (i) *targets* ( $T_1, T_2, \dots$ ) related to different LVM systems, (ii) inertial sensors (two-axis inclinometer and compass), and (iii) a tip in contact with the point of interest.



## Goal

This research presents a new mathematical model to localize the probe tip ( $P$ ) in measurements involving combinations of LVM systems.

The model takes the following factors into account:

- Position/orientation of any distributed sensor.
- Number, typology and relative position of probe targets.
- Metrological features of the sensors in use.

## Description

An overdefined system of equations is *linearized* and solved through the method of *Generalized Least Squares* (GLS), which can be used to give greater weight to the equations producing less uncertainty and *vice versa*. For the model to be viable, some parameters relating to the sensors should be known in advance, e.g., uncertainties in the position/orientation or local measurements; this can be done through *ad hoc* experimental tests or using manuals or technical documentation of the LVM systems in use.

The proposed model can also be used to estimate the uncertainty in the probe-tip localization: a covariance matrix can be determined by applying the *Multivariate Law of Propagation of Uncertainty* (MLPU) to the system of equations.

The model is automatable and could be a key tool to promote the combined use of LVM systems.

(1) A linearized system of equations,  $A \cdot X - B = 0$ , includes:

- one eq. for each pair of *distance* sensor and probe target;
- two eqs. for each pair of *angular* sensor and probe target;
- three eqs. related to a two-axis inclinometer and compass integrated into the probe.

(2) GLS method:

$$\hat{X} = (A^T \cdot W \cdot A)^{-1} \cdot A^T \cdot W \cdot B$$

being  $W$  a matrix that takes into account the uncertainty of the equations (which depends on the metrological features of the sensors in use).

(3) Estimate of the uncertainty in the localization of  $P$ :

$$\Sigma_X = (A^T \cdot W \cdot A)^{-1}$$

Italian provisional patent application no. 102016000107650, filed on 25/10/2016: Maisano, D., Mastrogiacommo, L., Galetto, M., Franceschini, F. *Dispositivo tastatore e procedimento di localizzazione di un dispositivo tastatore per la misura di oggetti di grandi dimensioni*.

## Applications

- Aerospace, Railway and Marine Industry**
  - dimensional verification
  - assembly
- Construction** of large-sized technological structures, such as wind turbines, tanks, etc.

## Advantages

- Better use of the instruments available;
- Adaptable to any combination of LVM instruments;
- Modular structure, easy to configure and customize;
- Efficient and effective localization procedure, suitable to real-time applications.

## Typical LVM instruments



**Acknowledgements:** This research was supported by the project Co-LVM “Cooperative multi-sensor data fusion for enhancing Large-Volume Metrology applications”, financed by Fondazione CRT under the initiative “La Ricerca dei Talenti”.